For Supervisor's use only

3

90636

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2006

90636 Integrate functions and use integrals to solve problems

Credits: Six
9.30 am Wednesday 29 November 2006

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show the results of any integration needed to solve the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**	
Integrate functions and use integrals to solve problems. ☐	Use advanced integration techniques to find integrals and solve problems. ☐	Solve more complex integration problem(s). ☐	
Overall Level of Performance ☐			

You are advised to spend 50 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

Find the integrals.
You do not need to simplify your answers.
Do not forget the arbitrary constant.

(a) $\int \sec 3x \tan 3x \, dx$

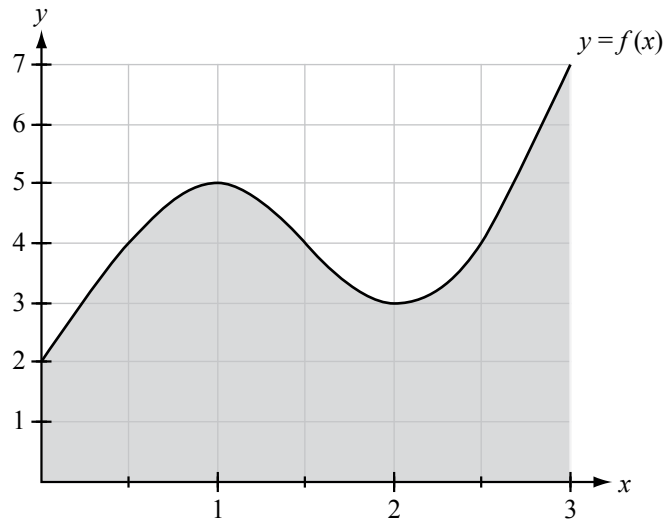
(b) $\int 4e^{5x+2} \, dx$

(c) $\int \frac{\sqrt{x} + 3x - 2}{x} \, dx$

QUESTION TWO

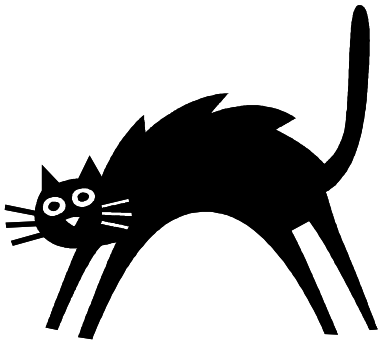
Assessor's use only

Use the trapezium rule to estimate the value of $\int_0^3 f(x)\,dx$ using six sub-intervals as shown by the shaded area below.



QUESTION THREE

Assessor's use only



A scared cat runs in a straight line along the top of a wall from one end in such a way that its velocity at time t is given by:

$v = 3 - 3 \sin 3t, \quad 0 \leq t \leq 5$

where v = velocity in metres per second
and t = time in seconds.

Calculate the distance travelled by the cat during the first four seconds.

Show the results of any integration needed to solve the problem.

QUESTION FOUR

Assessor's use only

Find the integral:

$\int \frac{x}{\sqrt{x+2}}\,\mathrm{d}x$ A suitable substitution may be helpful.

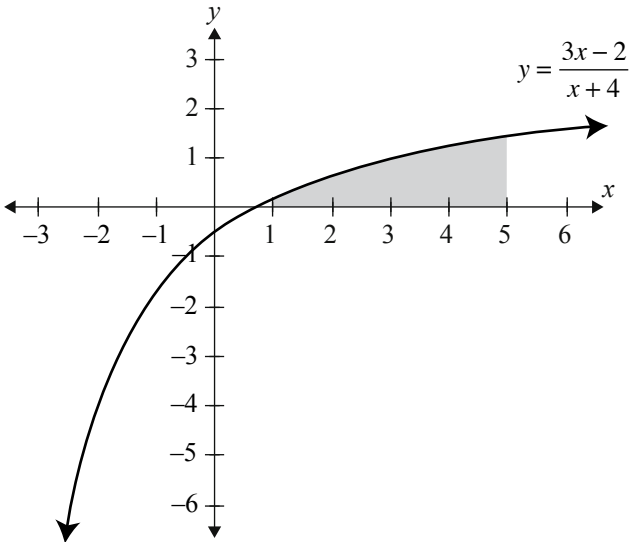
QUESTION FIVE

Assessor's use only

Use integration to calculate the shaded area enclosed by the curve $y = \dfrac{3x-2}{x+4}$ and the lines $y = 0$, $x = 1$ and $x = 5$.

Show your working.

Show the results of any integration needed to solve the problem.



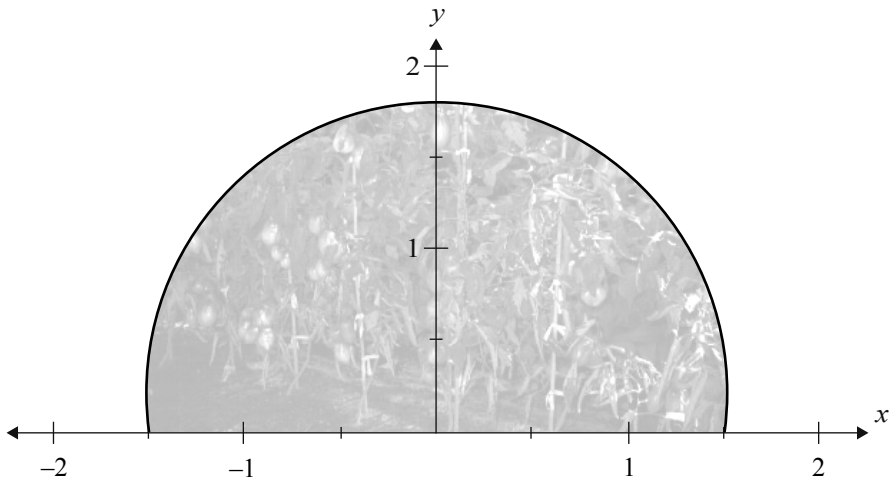
QUESTION SIX

Assessor's use only

A hothouse for plants has the shape of part of a sphere.

This shape can be obtained by revolving part of the circle

$x^2 + (y - 0.3)^2 = 2.25$ about the y-axis.



Calculate the volume, in m^3, contained within the hothouse.

Show the results of any integration needed to solve the problem.

QUESTION SEVEN

Assessor's use only

A wet porous substance in open air loses its moisture at a rate proportional to its moisture content.

A towel hung in the wind loses half its moisture during the first hour.

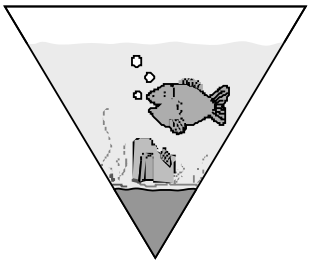
When will the towel have lost 99% of its moisture, assuming there is no change in weather conditions?

Write a differential equation and solve it to answer the question.

Show the results of any integration needed to solve the problem.

QUESTION EIGHT

Assessor's use only



A fish tank has a triangular cross-section.

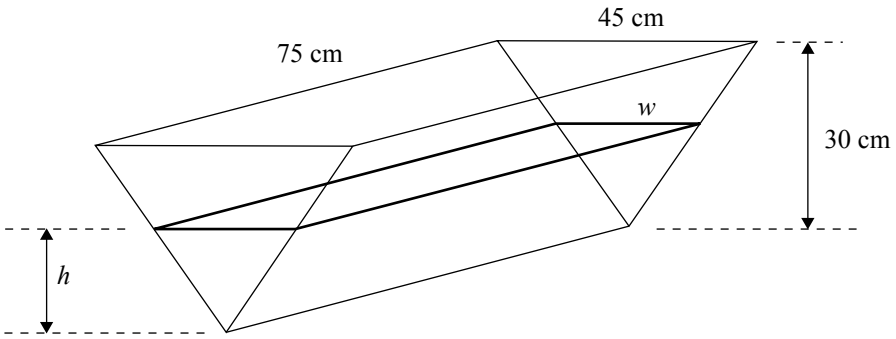
It has a length of 75 cm, width of 45 cm and depth of 30 cm.

Initially it is full of water.

After 5 days, the depth of the water has dropped to 28 cm.

The rate of evaporation is proportional to the surface area of the water.

Calculate how long the water in the fish tank takes to evaporate completely from when it was full.



Show the results of any integration needed to solve the problem.

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number	